

# Package ‘SpatEntropy’

April 7, 2021

**Type** Package

**Title** Spatial Entropy Measures

**Version** 2.0-1

**Author** L. Altieri, D. Cocchi, G. Roli

**Maintainer** Altieri Linda <linda.altieri@unibo.it>

**Depends** R (>= 3.5.0), spatstat (>= 2.0-0)

**Imports** spatstat.geom, spatstat.core, spatstat.linnet

**Description** The heterogeneity of spatial data presenting a finite number of categories can be measured via computation of spatial entropy. Functions are available for the computation of the main entropy and spatial entropy measures in the literature. They include the traditional version of Shannon's entropy (Shannon, 1948 <doi:10.1002/j.1538-7305.1948.tb01338.x>), Batty's spatial entropy (Batty, 1974 <doi:10.1111/j.1538-4632.1974.tb01014.x>), O'Neill's entropy (O'Neill et al., 1998 <doi:10.1007/BF00162741>), Li and Reynolds' contagion index (Li and Reynolds, 1993 <doi:10.1007/BF00125347>), Karlstrom and Ceccato's entropy (Karlstrom and Ceccato, 2002 <urn:nbn:se:kth:diva-61351>), Leibovici's entropy (Leibovici, 2009 <doi:10.1007/978-3-642-03832-7\_24>), Parresol and Edwards' entropy (Parresol and Edwards, 2014 <doi:10.3390/e16041842>) and Altieri's entropy (Altieri et al., 2018, <doi:10.1007/s10651-017-0383-1>). Full references for all measures can be found under the topic 'SpatEntropy'. The package is able to work with lattice and point data. The updated version works with the updated 'spatstat' package (>= 2.0-0). It also provides a more intuitive framework for all functions, including improved examples, and new data. The speed of most functions has also been substantially increased.

**License** GPL-3

**Encoding** UTF-8

**LazyData** true

**RoxygenNote** 7.1.1

**NeedsCompilation** no

**Repository** CRAN

**Date/Publication** 2021-04-07 09:50:02 UTC

## R topics documented:

altieri	2
areapart	4
batty	6
contagion	8
karlstrom	9
leibovici	11
oneill	13
parredw	14
raintrees	15
raintreesCOV	17
shannon	18
shannonZ	19
SpatEntropy	21
turin	22
turinTess	23
turinW	24
varshannon	25
varshannonZ	26
<b>Index</b>	<b>28</b>

---

altieri	<i>Altieri's spatial entropy.</i>
---------	-----------------------------------

---

### Description

This function computes spatial mutual information and spatial residual entropy as in Altieri et al (2017) and following works. References can be found at SpatEntropy.

### Usage

```
altieri(data, cell.size = 1, distbreak = "default", verbose = F)
```

### Arguments

data	If data are lattice, a data matrix, which can be numeric, factor, character, ... If the dataset is a point pattern, data is a ppp object.
cell.size	A single number. If data are lattice, the length of the side of each pixel. Default to 1. Ignored if data are points.
distbreak	Numeric. The chosen distance breaks for selecting pairs of pixels/points within the observation area. The default option is <code>c(cell.size, 2*cell.size)</code> for lattice data, and <code>c(mindist, 2*mindist)</code> for point data, where <code>mindist</code> is one tenth of the maximum distance within the observation area. Only the internal breaks have to be specified, the first and last break are automatically added as 0 and the maximum distance within the observation area, respectively.
verbose	Logical. If TRUE an output is printed in order to follow the progress of the work (recommended for large dataset). Default set to FALSE.

## Details

The computation of Altieri's entropy starts from a point or areal dataset, for which Shannon's entropy of the transformed variable  $Z$  (for details see [shannonZ](#))

$$H(Z) = \sum p(z_r) \log(1/p(z_r))$$

is computed using all possible pairs within the observation area. Then, its two components spatial mutual information

$$SMI(Z, W) = \sum p(w_k) \sum p(z_r|w_k) \log(p(z_r|w_k)/p(z_r))$$

and spatial residual entropy

$$H(Z)_W = \sum p(w_k) \sum p(z_r|w_k) \log(1/p(z_r|w_k))$$

are calculated in order to account for the overall role of space in determining the data heterogeneity. Besides, starting from a partition into distance classes, a list of adjacency matrices is built, which identifies what pairs of units must be considered for each class. Spatial mutual information and spatial residual entropy are split into local terms according to the chosen distance breaks, so that the role of space can be investigated both in absolute and relative terms. In the function output, the relative partial terms are returned so that they sum to 1 for each distance class: e.g. if the relative SPI terms is 0.3 and the relative residual term is 0.7, the interpretation is that, at the specific distance class, 30% of the entropy is due to the role of space as a source of heterogeneity. The function is able to work with lattice data with missing data, as long as they are specified as NAs: missing data are ignored in the computations. The function is able to work with grids containing missing data, specified as NA values. All NAs are ignored in the computation and only couples of non-NA observations are considered.

## Value

A list with elements:

- `distance.breaks` a two column matrix with the lower and upper extreme of each distance class
- `SPI.terms` the spatial partial information terms
- `rel.SPI.terms` the relative version of spatial partial information terms (see the details)
- `RES.terms` the spatial partial residual entropies
- `rel.RES.terms` the relative version of spatial partial residual entropies (see the details)
- `SMI` the spatial mutual information
- `RES` the global residual entropy
- `ShannonZ` Shannon's entropy of  $Z$  in the same format as the output of [shannonZ\(\)](#)
- `W.distribution` the spatial weights for each distance range
- `total.pairs` the total number of pairs over the area (realizations of  $Z$ )
- `class.pairs` the number of pairs for each distance range.
- `cond.Z.distribution` a list with the conditional absolute and relative frequencies of  $Z$  for each distance range

## Examples

```

#lattice data
data=matrix(sample(1:5, 100, replace=TRUE), nrow=10)
outp=altieri(data)
outp=altieri(data, cell.size=2) #same result
outp=altieri(data, cell.size=2, distbreak=c(2, 5))
#plot data
plot(as.im(data, W=square(nrow(data))),
      col=gray(seq(1,0,l=length(unique(c(data)))))),
      main="", ribbon=TRUE)

#lattice data with missing values
data=matrix(sample(1:5, 100, replace=TRUE), nrow=10)
data=rbind(rep(NA, ncol(data)), data, rep(NA, ncol(data)))
outp=altieri(data)
#plot data
plot(as.im(data, W=square(nrow(data))),
      col=topo.colors(length(unique(c(data)[!is.na(c(data))])),
                      main="", ribbon=TRUE)

#point data
data=ppp(x=runif(400), y=runif(400), window=square(1),
         marks=(sample(c("a","b","c"), 400, replace=TRUE)))
outp=altieri(data)
outp=altieri(data, verbose=TRUE)
#plot data
plot(data, cols=1:length(unique(marks(data))), main="", pch=16)
#check what happens for badly specified distance breaks
#outp=altieri(data, distbreak=c(1,1.4))
#outp=altieri(data, distbreak=c(1,2))

```

---

areapart

*Area partition.*

---

## Description

This function partitions the observation area in a number of sub-areas, and assigns the data points/pixels to the areas. This function is useful either when a random partition wants to be created, or when the user wants to set the area's centroids and is happy with an area tessellation in Voronoi polygons according to the defined centroids.

## Usage

```
areapart(data, G, cell.size = 1)
```

**Arguments**

data	If data are lattice, a data matrix, which can be numeric, factor, character, ... If the dataset is a point pattern, data is a ppp object.
G	An integer if sub-areas are randomly generated, determining the number $G$ of sub-areas. Alternatively, a 2-column matrix with the sub-areas centroids' coordinates.
cell.size	A single number. If data are lattice, the length of the side of each pixel. Default to 1. Ignored if data are points.

**Details**

The function is preliminary to the computation of Batty's or Karlstrom and Ceccato's entropy. An event of interest (in the form of a point or binary areal dataset) occurs over an observation area divided into sub-areas. If the partition is random, this function generates the sub-areas by randomly drawing the areas' centroids over the observation window. Then, data points/pixels are assigned to the area with the closest centroid. When data are pixels, each pixel is assigned to an area according to the coordinates of its own centroid. The function also works for non-binary datasets and marked ppp objects.

**Value**

A list with elements:

- G.pp a point pattern containing the  $G$  areas' centroids
- data.assign a four column matrix, with all pairs of data coordinates and data values matched to one of the  $G$  areas (numbered 1 to  $G$ ). If the dataset is an unmarked ppp object, the data category column is a vector of 1s.

**Examples**

```
#LATTICE DATA

data=matrix(sort(sample(c("a","b","c"), 100, replace=TRUE)), nrow=10)
partition=areapart(data, G=5)
partition=areapart(data, G=5, cell.size=2)
##to plot
#data
plot(as.im(data, W=partition$G.pp>window), main="",
      col=gray(seq(1,0,l=length(unique(c(data)[!is.na(c(data))])))))
#partition centroids
plot(partition$G.pp, add=TRUE, pch=16, col=2)
#partition tessellation
plot(dirichlet(partition$G.pp), add=TRUE, border=2)

#providing a pre-fixed area partition
data=matrix(sort(sample(c("a","b","c"), 100, replace=TRUE)), nrow=10)
win=square(nrow(data))
GG=cbind(runif(5, win$xrange[1], win$xrange[2]),
         runif(5, win$yrange[1], win$yrange[2]))
partition=areapart(data, G=GG)
```

```
#POINT DATA

data=ppp(x=runif(100), y=runif(100), window=square(1))
partition=areapart(data, 10)
#to plot
plot(data)
plot(partition$G.pp, add=TRUE, col=2, pch=16)
plot(dirichlet(partition$G.pp), add=TRUE, border=2)

#with marks
data=ppp(x=runif(100), y=runif(100), window=square(1),
        marks=(sample(c("a","b","c"), 100, replace=TRUE)))
GG=cbind(runif(10, data$window$xrange[1], data$window$xrange[2]),
        runif(10, data$window$yrange[1], data$window$yrange[2]))
partition=areapart(data, G=GG)
```

---

batty

*Batty's entropy.*


---

## Description

This function computes Batty's spatial entropy, following Batty (1976), see also Altieri et al (2017 and following) (references are under the topic [SpatEntropy](#)).

## Usage

```
batty(data, category = 1, cell.size = 1, partition = 10)
```

## Arguments

data	If data are lattice, a data matrix, which can be numeric, factor, character, ... If the dataset is a point pattern, data is a ppp object.
category	A single value matching the data category of interest for computing Batty's entropy. Default to 1. If the dataset is an unmarked point pattern, this argument must not be changed from the default.
cell.size	A single number. If data are lattice, the length of the side of each pixel. Default to 1. Ignored if data are points.
partition	Input defining the partition into subareas. If an integer, it defines the number of sub-areas that are randomly generated by <a href="#">areapart</a> ; if a two column matrix with coordinates, they are the centroids of the subareas built by <a href="#">areapart</a> . Alternatively, it can be the output of <a href="#">areapart</a> , or a tess object built by the user. The default option is <code>partition=areapart(data,G=10)</code> , which generates 10 random sub-areas.

## Details

Batty's spatial entropy measures the heterogeneity in the spatial distribution of a phenomenon of interest, with regard to an area partition. It is high when the phenomenon is equally intense over the sub-areas, and low when it concentrates in one or few sub-areas. This function allows to compute Batty's entropy as

$$H_B = \sum p_g \log(T_g/p_g)$$

where  $p_g$  is the probability of occurrence of the phenomenon over sub-area  $g$ , and  $T_g$  is the sub-area size. When data are categorical, the phenomenon of interest corresponds to one category, which must be specified. If data are an unmarked point pattern, a fake mark vector is created with the same category for all points. For comparison purposes, the relative version of Batty's entropy is also returned, i.e. Batty's entropy divided by its maximum  $\log(\sum T_g)$ . Note that when the total observation area is 1, then  $\log(\sum T_g) = 0$ , therefore in that case during the computation all  $T_g$ s are multiplied by 100 and a warning is produced. The function is able to work with grids containing missing data, specified as NA values. All NAs are ignored in the computation.

## Value

A list of four elements:

- `area.tess` a tess object with the area partition
- `areas.freq` a dataframe giving, for each sub-area, the absolute and relative frequency of the points/pixels of interest and the sub-area size for each sub-area
- `batty` Batty's entropy
- `rel.batty` Batty's entropy divided by  $\log(\sum T_g)$  for comparison across observation areas.

## Examples

```
#LATTICE DATA

data=matrix((sample(c("a","b","c"), 100, replace=TRUE)), nrow=10)
batty.entropy=batty(data, category="a")
##to plot
data.binary=matrix(as.numeric(data=="a"), nrow(data))
plot(as.im(data.binary, W=batty.entropy$areas.tess$window), main="",
      col=gray(seq(1,0,l=length(unique(c(data.binary))))), ribbon=FALSE)
plot(batty.entropy$areas.tess, add=TRUE, border=2)

#POINT DATA

#unmarked pp
data=ppp(x=runif(100, 0, 10), y=runif(100, 0, 10), window=square(10))
batty.entropy=batty(data)
##to plot
plot(data)
plot(batty.entropy$areas.tess, add=TRUE, border=2)

#marked pp
data=ppp(x=runif(100, 0, 10), y=runif(100, 0, 10), window=square(10),
        marks=(sample(1:5, 100, replace=TRUE)))
```

```

plot(data) #see ?plot.ppp for options
#if you want to compute the entropy on all points
batty.entropy=batty(unmark(data))
#if you want to compute the entropy on a category, say 3
batty.entropy=batty(data, category=3)
##to plot using the selected category
ind=which(marks(data)==3)
data.binary=unmark(data[ind])
plot(data.binary)
plot(batty.entropy$areas.tess, add=TRUE, border=2)

```

---

contagion

*Li and Reynolds' relative contagion index.*


---

### Description

This function computes Li and Reynold's contagion index, following Li and Reynolds (1993), starting from a data matrix. References can be found at [SpatEntropy](#).

### Usage

```
contagion(data)
```

### Arguments

data                    A data matrix or vector, can be numeric, factor, character, ...

### Details

This index is based on the transformed variable  $Z$  identifying couples of realizations of the variable of interest. A distance of interest is fixed: the contagion index is originally thought for areas sharing a border, as O'Neill's entropy. Then, all contiguous couples of realizations of the variable of interest are counted and their relative frequencies are used to compute the index, which is  $1 - NO$  where  $NO$  is the relative version of O'Neill's entropy, i.e. O'Neill's entropy divided by its maximum  $\log(I^2)$ ,  $I$  being the number of categories of the variable under study. The relative contagion index ranges from 0 (no contagion, maximum entropy) to 1 (maximum contagion). The function is able to work with grids containing missing data, specified as NA values. All NAs are ignored in the computation and only couples of non-NA observations are considered.

### Value

a list of two elements:

- probabilities - a table with the estimated probabilities (relative frequencies) for all couple categories;
- contagion - Li and Reynold's contagion index.



## Examples

```
#numeric data, square grid
data=matrix(sample(1:5, 100, replace=TRUE), nrow=10)
contagion(data)
#plot data
plot(as.im(data, W=square(nrow(data))),
     col=gray(seq(1,0,l=length(unique(c(data)))))),
     main="", ribbon=TRUE)

#character data, rectangular grid
data=matrix(sample(c("a","b","c"), 300, replace=TRUE), nrow=30)
contagion(data)
#plot data
plot(as.im(data, W=owin(xrange=c(0,ncol(data)), yrange=c(0,nrow(data)))),
     col=terrain.colors(length(unique(c(data))))),
     main="", ribbon=TRUE)
```

---

karlstrom

*Karlstrom and Ceccato's entropy.*


---

## Description

This function computes Karlstrom and Ceccato's spatial entropy for a chosen neighbourhood distance, following Karlstrom and Ceccato (2002), see also Altieri et al (2017) and following works (references are under the topic [SpatEntropy](#)).

## Usage

```
karlstrom(
  data,
  category = 1,
  cell.size = 1,
  partition = 10,
  neigh = 4,
  method = "number"
)
```

## Arguments

data	If data are lattice, a data matrix, which can be numeric, factor, character, ... If the dataset is a point pattern, data is a ppp object.
category	A single value matching the data category of interest for computing Batty's entropy. Default to 1. If the dataset is an unmarked point pattern, this argument must not be changed from the default.
cell.size	A single number. If data are lattice, the length of the side of each pixel. Default to 1. Ignored if data are points.

partition	Input defining the partition into subareas. If an integer, it defines the number of sub-areas that are randomly generated by <code>areapart</code> ; if a two column matrix with coordinates, they are the centroids of the subareas built by <code>areapart</code> . Alternatively, it can be the output of <code>areapart</code> , or a tess object built by the user. The default option is <code>partition=areapart(data,G=10)</code> , which generates 10 random sub-areas.
neigh	A single number. It can be either the number of neighbours for each sub-area (including the area itself). or the Euclidean distance to define which sub-areas are neighbours, based on their centroids. Default to 4 neighbours.
method	Character, it guides the interpretation of <code>neigh</code> . Either "number" (the default) or "distance".

### Details

Karlstrom and Ceccato's spatial entropy measures the heterogeneity in the spatial distribution of a phenomenon of interest, with regard to an area partition and accounting for the neighbourhood. It is similar to Batty's entropy (see [batty](#)) discarding the sub-area size, with the difference that the probability of occurrence of the phenomenon over area  $g$  is actually a weighted sum of the neighbouring probabilities.

$$H_{KC} = \sum p_g \log(1/\tilde{p}_g)$$

where  $p_g$  is the probability of occurrence of the phenomenon over sub-area  $g$ , and  $\tilde{p}_g$  is the averaged probability over the neighbouring areas (including the  $g$ -th area itself). When data are categorical, the phenomenon of interest corresponds to one category, which must be specified. If data are an unmarked point pattern, a fake mark vector is created with the same category for all points. For comparison purposes, the relative version of Karlstrom and Ceccato's entropy is also returned, i.e. Karlstrom and Ceccato's entropy divided by its maximum  $\log(\text{number of sub-areas})$ . The function is able to work with grids containing missing data, specified as NA values. All NAs are ignored in the computation.

### Value

A list of four elements:

- `area.tess` a tess object with the area partition
- `areas.freq` a dataframe giving, for each sub-area, the absolute and relative frequency of the points/pixels of interest, the weighted probabilities of the neighbours and the sub-area size for each sub-area
- `karlstrom` Karlstrom and Ceccato's entropy
- `rel.karl` Karlstrom and Ceccato's entropy divided by  $\log(G)$  (number of sub-areas) for comparison across observation areas.

### Examples

```
#LATTICE DATA

data=matrix((sample(c("a","b","c"), 100, replace=TRUE)), nrow=10)
KC.entropy=karlstrom(data, category="a")
KC.entropy=karlstrom(data, category="a", neigh=3.5, method="distance")
```

```

##to plot
data.binary=matrix(as.numeric(data=="a"), nrow(data))
plot(as.im(data.binary, W=KC.entropy$areas.tess>window), main="",
      col=gray(seq(1,0,l=length(unique(c(data.binary))))), ribbon=FALSE)
plot(KC.entropy$areas.tess, add=TRUE, border=2)

#POINT DATA

#unmarked pp
data=ppp(x=runif(100, 0, 10), y=runif(100, 0, 10), window=square(10))
KC.entropy=karlstrom(data)
##to plot
plot(data)
plot(KC.entropy$areas.tess, add=TRUE, border=2)

#marked pp
data=ppp(x=runif(100, 0, 10), y=runif(100, 0, 10), window=square(10),
         marks=(sample(1:5, 100, replace=TRUE)))
#if you want to compute the entropy on all points
KC.entropy=karlstrom(unmark(data))
#if you want to compute the entropy on a category, say 3
KC.entropy=karlstrom(data, category=3)
##to plot using the selected category
ind=which(marks(data)==3)
data.binary=unmark(data[ind])
plot(data.binary)
plot(KC.entropy$areas.tess, add=TRUE, border=2)

```

---

leibovici

*Leibovici's entropy.*


---

## Description

This function computes Leibovici's entropy according to a chosen distance  $d$  (with O'Neill's entropy as a special case) following Leibovici (2009), see also Altieri et al (2017). References can be found at [SpatEntropy](#).

## Usage

```
leibovici(data, cell.size = 1, ccdist = cell.size, verbose = F)
```

## Arguments

<code>data</code>	If data are lattice, a data matrix, which can be numeric, factor, character, ... If the dataset is a point pattern, data is a ppp object.
<code>cell.size</code>	A single number. If data are lattice, the length of the side of each pixel. Default to 1. Ignored if data are points.

ccdist	A single number. The chosen distance for selecting couples of pixels/points within the observation area. Default to <code>cell.size</code> .
verbose	Logical. If TRUE an output is printed in order to follow the progress of the work (recommended for large dataset). Default set to FALSE.

### Details

This index is based on the transformed variable  $Z$  identifying couples of realizations of the variable of interest. A distance of interest is fixed, which in the case of O'Neill's entropy is the contiguity, i.e. sharing a border for lattice data. Then, all couples of realizations of the variable of interest lying at a distance smaller or equal to the distance of interest are counted, and their relative frequencies are used to compute the index with the traditional Shannon's formula. #'

$$H_L(Z) = \sum p(z_r|d) \log(1/p(z_r|d))$$

where  $z_r = (x_i, x_{i'})$  is a generic couple of realizations of the study variable  $X$ . The conditioning on  $d$  means that only couples within a predefined distance are considered. The maximum value for Leibovici's entropy is  $\log(I^2)$  where  $I$  is the number of categories of the study variable  $X$ . The relative version of Leibovici's entropy is obtained by dividing the entropy value by its maximum, and is useful for comparison across datasets with a different number of categories. The function is able to work with grids containing missing data, specified as NA values. All NAs are ignored in the computation and only couples of non-NA observations are considered.

### Value

a list of three elements:

- `probabilities` - a table with the estimated probabilities (relative frequencies) for all couple categories;
- `leib` - Leibovici's entropy;
- `rel.leib` - Leibovici's relative entropy.

### Examples

```
#random grid data - high entropy
data=matrix(sample(c("a","b","c"), 400, replace=TRUE), nrow=20)
leibovici(data, cell.size=1, ccdist=2)
#plot data
plot(as.im(data, W=square(nrow(data))),
      col=gray(seq(1,0,l=length(unique(c(data))))),
      main="", ribbon=TRUE)

#compact grid data - low entropy
data=matrix(sort(sample(c("a","b","c"), 400, replace=TRUE)), nrow=20)
leibovici(data, cell.size=1, ccdist=1.5)
#plot data
plot(as.im(data, W=square(nrow(data))),
      col=heat.colors(length(unique(c(data))))),
      main="", ribbon=TRUE)
```

```
#point data
data=ppp(x=runif(400), y=runif(400), window=square(1), marks=sample(1:4, 400, replace=TRUE))
leibovici(data, ccdist=0.1)
#plot data
plot(data)
#nicer plot
data.cat=data; marks(data.cat)=as.character(marks(data))
plot(data.cat, cols=1:length(unique(marks(data.cat))), main="", pch=16)
```

---

oneill

*O'Neill's entropy.*


---

## Description

This function computes O'Neill's entropy for a data matrix (see O'Neill et al, 1988).

## Usage

```
oneill(data)
```

## Arguments

data                    A data matrix, can be numeric, factor, character, ...

## Details

O'Neill's entropy index is based on the transformed variable  $Z$ , identifying couples of realizations of the variable of interest:

$$H_O(Z) = \sum p(z_r|C) \log(1/p(z_r|C))$$

where  $z_r = (x_i, x_{i'})$  is a generic couple of realizations of the study variable  $X$ . The conditioning on  $C$  for grid data means that only contiguous couples are considered, i.e. couples of pixels sharing a border. All contiguous couples of realizations of the variable of interest are counted and their relative frequencies are used to compute the index. The maximum value for O'Neill's entropy is  $\log(I^2)$  where  $I$  is the number of categories of  $X$ . The relative version of O'Neill's entropy is obtained by dividing the entropy value by its maximum, and is useful for comparison across datasets with a different number of categories. The function is able to work with grids containing missing data, specified as NA values. All NAs are ignored in the computation and only couples of non-NA observations are considered.

## Value

a list of three elements:

- probabilities - a table with the estimated probabilities (relative frequencies) for all couple categories;
- oneill - O'Neill's entropy;
- rel.oneill - O'Neill's relative entropy.

**Examples**

```
#numeric data, square grid
data=matrix(sample(1:5, 100, replace=TRUE), nrow=10)
oneill(data)
#plot data
plot(as.im(data, W=square(nrow(data))),
      col=gray(seq(1,0,l=length(unique(c(data))))),
      main="", ribbon=TRUE)

#character data, rectangular grid
data=matrix(sample(c("a","b","c"), 300, replace=TRUE), nrow=30)
oneill(data)
#plot data
plot(as.im(data, W=owin(xrange=c(0,ncol(data)), yrange=c(0,nrow(data))),
      col=terrain.colors(length(unique(c(data))))),
      main="", ribbon=TRUE)

#data with missing values
data=matrix(sample(1:5, 100, replace=TRUE), nrow=10)
data=rbind(rep(NA, ncol(data)), data, rep(NA, ncol(data)))
oneill(data)
```

---

 parredw

*Parresol and Edwards' entropy.*


---

**Description**

Compute Parresol and Edwards' entropy, following Parresol and Edwards (2014), starting from data. References can be found at [SpatEntropy](#).

**Usage**

```
parredw(data)
```

**Arguments**

data                    A data matrix or vector, can be numeric, factor, character, ...

**Details**

This index is based on the transformed variable  $Z$  identifying couples of realizations of the variable of interest. A distance of interest is fixed: Parresol and Edwards' entropy is thought for areas sharing a border, as O'Neill's entropy. All contiguous couples of realizations of the variable of interest are counted and their relative frequencies are used to compute the index, which is the opposite of O'Neill's entropy. The function is able to work with grids containing missing data, specified as NA values. All NAs are ignored in the computation and only couples of non-NA observations are considered.

**Value**

a list of two elements:

- `probabilities` - a table with the estimated probabilities (relative frequencies) for all couple categories;
- `parredw` - Parresol and Edwards' entropy.

**Examples**

```
#numeric data, square grid
data=matrix(sample(1:5, 100, replace=TRUE), nrow=10)
parredw(data)
#plot data
plot(as.im(data, W=square(nrow(data))),
      col=gray(seq(1,0,l=length(unique(c(data))))),
      main="", ribbon=TRUE)

#character data, rectangular grid
data=matrix(sample(c("a","b","c"), 300, replace=TRUE), nrow=30)
parredw(data)
#plot data
plot(as.im(data, W=owin(xrange=c(0,ncol(data)), yrange=c(0,nrow(data))),
      col=terrain.colors(length(unique(c(data))))),
      main="", ribbon=TRUE)
```

---

raintrees

*Rainforest tree data.*

---

**Description**

A marked point pattern dataset about four rainforest tree species.

**Usage**

```
raintrees
```

**Format**

A ppp object (see package spatstat) with 7251 points, containing:

**window** An object of type `owin` (see package spatstat), the 1000x500 metres observation area

**x, y** Numeric vectors with points' coordinates

**marks** A character vector matching the tree species to the data points

## Details

This dataset documents the presence of tree species over Barro Colorado Island, Panama. Barro Colorado Island has been the focus of intensive research on lowland tropical rainforest since 1923 (<http://www.ctfs.si.edu>). Research identified several tree species over a rectangular observation window of size 1000x500 metres; the tree species constitute the point data categorical mark. This dataset presents 4 species with different spatial configurations: *Acalypha diversifolia*, *Chamguava schippii*, *Inga pezizifera* and *Rinorea sylvatica*. The overall dataset has a total number of 7251 points. The dataset is analyzed with spatial entropy measures in Altieri et al (2018) (references can be found at [SpatEntropy](#)).

## Source

<http://www.ctfs.si.edu>

## Examples

```
data(raintrees)
#plot(raintrees, main="", pch=16, cols=1:4)

#shannon's entropy of the four trees
shannon(raintrees)

#shannon's entropy of Z (tree pairs)
shannonZ(raintrees)

#leibovici's entropy
raintrees$window #to check size and unit of measurement
#example run on a subset of the data to speed up computations
subdata=raintrees[owin(c(0,200),c(0,100))]; plot(subdata)
outp=leibovici(subdata, ccdist=10, verbose=TRUE)
#do not worry about warnings like "data contain duplicated points": since
#coordinates are rounded to the first decimal place, it looks like
#some trees are overlapping when they are just very close.
#Entropy computation works properly anyway

#altieri's entropy
#example run on a subset of the data to speed up computations
subdata=raintrees[owin(c(0,200),c(0,100))]; plot(subdata)
outp=altieri(subdata, distbreak=c(1,2,5,10), verbose=TRUE)

#batty's entropy
#on all points, with a random partition in 10 sub-areas
batty.ent=batty(unmark(raintrees), partition=10)
#plot with partition
#plot(unmark(raintrees), pch=16, cex=0.6, main="")
#plot(batty.ent$areas.tess, add=TRUE, border=2, lwd=2)
#on a specific tree species, with a random partition in 6 sub-areas
unique(marks(raintrees)) #to check the species' names
#plot(split.ppp(raintrees), main="") #to plot by species
batty.ent=batty(raintrees, category="cha2sc", partition=6)
#plot with partition
```



```

#plot(split.ppp(raintrees)$cha2sc, pch=16, cex=0.6, main="")
#plot(batty.ent$areas.tess, add=TRUE, border=2)

#batty's entropy with a partition based on the covariate,
#exploiting spatstat functions
data(raintreesCOV)
#plot(raintreesCOV$grad, main="", col=gray(seq(1,0,l=100)))
data=split.ppp(raintrees)$acaldi
#plot(data, add=TRUE, pch=16, cex=0.6, main="")
#discretize the covariate
slopecut=cut(raintreesCOV$grad,
             breaks = quantile(raintreesCOV$grad, probs = (0:4)/4),
             labels = 1:4)
maskv=tiles=list()
for(ii in 1:nlevels(slopecut))
{
maskv[[ii]]=as.logical(c(slopecut$v)==levels(slopecut)[ii])
tiles[[ii]]=owin(xrange=data$window$xrange,
                 yrange=data$window$yrange,
                 mask=matrix(maskv[[ii]],nrow(slopecut$v)))
}
slopetess=list(tiles=tiles, n=nlevels(slopecut))
#plot(slopecut, main = "", col=gray(seq(1,0.4,l=4)))
#plot(data, add=TRUE, pch=16, cex=0.6, main="", col=1)

batty(data, partition=slopetess)

#karlstrom and ceccato's entropy
#on a specific tree species, with a random partition in 6 sub-areas
unique(marks(raintrees)) #to check the species' names
#plot(split.ppp(raintrees), main="") #to plot by species
KC.ent=karlstrom(raintrees, category="rinosy", partition=6, neigh=3)
#plot with partition
#plot(split.ppp(raintrees)$rinosy, pch=16, cex=0.6, main="")
#plot(KC.ent$areas.tess, add=TRUE, border=2)

```

---

raintreesCOV

*Covariates for the rainforest tree data.*


---

## Description

A list of two pixel images with covariates altitude and soil slope for the rainforest tree species.

## Usage

```
raintreesCOV
```

**Format**

A list of two elements:

**elev** An object of type `im` (see package `spatstat`), the soil elevation

**grad** An object of type `im` (see package `spatstat`), the soil slope (gradient of elevation)

**Details**

For details of the dataset, see [raintrees](#). This accompanying dataset gives information about the elevation in the study region. It is a list containing two pixel images, `elev` (elevation in metres) and `grad` (norm of elevation gradient). These pixel images are objects of class `im`. Covariate values are continuous. Once discretized as wished, they can turn into categorical datasets for the computation of all entropy measures. Moreover, they can be used to build sensible sub-areas for Batty's and Karlstrom and Ceccato's entropies (see the examples).

**Source**

<http://www.ctfs.si.edu>

**Examples**

```
data(raintreesCOV)
plot(raintreesCOV, main="")

#shannon entropy
shannon(raintrees)
```

---

shannon

*Shannon's entropy.*

---

**Description**

This function computes Shannon's entropy of a variable  $X$  with a finite number of categories. Shannon's entropy is a non-spatial measure.

**Usage**

```
shannon(data)
```

**Arguments**

`data` A data matrix or vector, can be numeric, factor, character, ... Alternatively, a marked `ppp` object.

## Details

Shannon's entropy measures the heterogeneity of a set of categorical data. It is computed as

$$H(X) = \sum p(x_i) \log(1/p(x_i))$$

where  $p(x_i)$  is the probability of occurrence of the  $i$ -th category, here estimated, as usual, by its relative frequency. This is both the non parametric and the maximum likelihood estimator for entropy. Shannon's entropy varies between 0 and  $\log(I)$ ,  $I$  being the number of categories of the variable under study. The relative version of Shannon's entropy, i.e. the entropy divided by  $\log(I)$ , is also computed, under the assumption that all data categories are present in the dataset. The relative entropy is useful for comparison across datasets with different  $I$ . The function is able to work with lattice data with missing data, as long as they are specified as NAs: missing data are ignored in the computations.

## Value

a list of three elements:

- `probabilities` - a table with the estimated probabilities (relative frequencies) for all data categories;
- `shann` - Shannon's entropy;
- `rel.shann` - Shannon's relative entropy.

## Examples

```
#NON SPATIAL DATA
shannon(sample(1:5, 50, replace=TRUE))

#POINT DATA
#requires marks with a finite number of categories
data.pp=runifpoint(100, win=square(10))
marks(data.pp)=sample(c("a","b","c"), 100, replace=TRUE)
shannon(marks(data.pp))

#LATTICE DATA
data.lat=matrix(sample(c("a","b","c"), 100, replace=TRUE), nrow=10)
shannon(data.lat)
```

---

shannonZ

*Shannon's entropy of the transformed variable Z.*

---

## Description

This function computes Shannon's entropy of variable  $Z$ , where  $Z$  identifies pairs of realizations of the variable of interest.

**Usage**

```
shannonZ(data)
```

**Arguments**

`data` A data matrix or vector, can be numeric, factor, character, ... Alternatively, a marked ppp object.

**Details**

Many spatial entropy indices are based on the transformation  $Z$  of the study variable, i.e. on pairs (unordered couples) of realizations of the variable of interest. 'Unordered couples' means that the relative spatial location is irrelevant, i.e. that a couple where category  $i$  occurs at the left of category  $j$  is identical to a couple where category  $j$  occurs at the left of category  $i$ . When all possible pairs occurring within the observation areas are considered, Shannon's entropy of the variable  $Z$  may be computed as

$$H(Z) = \sum p(z_r) \log(1/p(z_r))$$

where  $p(z_r)$  is the probability of the  $r$ -th pair of realizations, here estimated by its relative frequency. Shannon's entropy of  $Z$  varies between 0 and  $\log(R)$ ,  $R = \text{binom}(n+1, 2)$  (where  $n$  is the number of observations) being the number of possible pairs of categories of the variable under study. The function is able to work with lattice data with missing data, as long as they are specified as NAs: missing data are ignored in the computations.

**Value**

a list of three elements:

- `probabilities` - a table with the estimated probabilities (relative frequencies) for all  $Z$  categories (data pairs);
- `shannZ` - Shannon's entropy of  $Z$ ;
- `rel.shannZ` - Shannon's relative entropy of  $Z$ .

**Examples**

```
#NON SPATIAL DATA
shannonZ(sample(1:5, 50, replace=TRUE))

#POINT DATA
data.pp=runifpoint(100, win=square(10))
marks(data.pp)=sample(c("a","b","c"), 100, replace=TRUE)
shannonZ(marks(data.pp))

#LATTICE DATA
data.lat=matrix(sample(c("a","b","c"), 100, replace=TRUE), nrow=10)
shannonZ(data.lat)
```

## Description

The heterogeneity of spatial data presenting a finite number of categories can be measured via computation of spatial entropy. Functions are available for the computation of the main entropy and spatial entropy measures in the literature. They include the traditional version of Shannon's entropy, Batty's spatial entropy, O'Neill's entropy, Li and Reynolds' contagion index, Karlstrom and Ceccato's entropy, Leibovici's entropy, Parresol and Edwards' entropy and Altieri's entropy. The package is able to work with lattice and point data.

## Details

### References:

- ALTIERI L., COCCHI D., ROLI G. (2019). Advances in spatial entropy measures. *Stochastic Environmental Research and Risk Assessment*
- ALTIERI L., COCCHI D., ROLI G. (2019). Measuring heterogeneity in urban expansion via spatial entropy. *Environmetrics*, 30(2), e2548
- ALTIERI L., COCCHI D., ROLI G. (2018). A new approach to spatial entropy measures. *Environmental and Ecological Statistics*, 25(1), 95-110
- Altieri, L., D. Cocchi, and G. Roli (2017). The use of spatial information in entropy measures. arXiv:1703.06001
- Batty, M. (1974). Spatial entropy. *Geographical Analysis* 6, 1-31.
- Batty, M. (1976). Entropy in spatial aggregation. *Geographical Analysis* 8, 1-21.
- EEA (2011). Corine land cover 2000 raster data. Technical Report, downloadable at <http://www.eea.europa.eu/data-and-maps/data/corine-land-cover-2000-raster-1>.
- Karlstrom, A. and V. Ceccato (2002). A new information theoretical measure of global and local spatial association. *The Review of Regional Research* 22, 13-40.
- Leibovici, D. (2009). Defining spatial entropy from multivariate distributions of co-occurrences. Berlin, Springer: In K. S. Hornsby et al. (eds.): 9th International Conference on Spatial Information Theory 2009, *Lecture Notes in Computer Science* 5756, 392-404.
- Li, H. and J. Reynolds (1993). A new contagion index to quantify spatial patterns of landscapes. *Landscape Ecology* 8(3), 155-162.
- O'Neill, R., J. Krummel, R. Gardner, G. Sugihara, B. Jackson, D. DeAngelis, B. Milne, M. Turner, B. Zygumt, S. Christensen, V. Dale, and R. Graham (1988). Indices of landscape pattern. *Landscape Ecology* 1(3), 153-162.
- Parresol, B. and L. Edwards (2014). An entropy-based contagion index and its sampling properties for landscape analysis. *Entropy* 16(4), 1842-1859.
- Shannon, C. (1948). A mathematical theory of communication. *Bell Dyditem Technical Journal* 27, 379-423, 623-656.

---

turin

*Turin urban data.*

---

### Description

A lattice dataset with Turin Urban Morphological Zones.

### Usage

turin

### Format

A matrix with 111 rows and 113 columns. Values are either 0 (non-urban) or 1 (urban). Pixels outside the administrative borders are classified as NA.

### Details

This raster/pixel/lattice dataset comes from the EU CORINE Land Cover project (EEA, 2011) and is dated 2011. It is the result of classifying the original land cover data into urbanised and non-urbanised zones, known as 'Urban Morphological Zones' (UMZ, see EEA, 2011). UMZ data are useful to identify shapes and patterns of urban areas, and thus to detect what is known as urban sprawl. Turin's metropolitan area is extracted from the European dataset and is composed by the municipality of Turin and the surrounding municipalities: Beinasco, Venaria Reale, San Mauro Torinese, Grugliasco, Borgaro Torinese, Collegno, Pecetto Torinese, Pino Torinese, Moncalieri, Nichelino, Settimo Torinese, Baldissero Torinese, Rivoli, Orbassano. The dataset is made of 111x113 pixels of size 250x250 metres.

### Source

EEA (2011). Corine land cover 2000 raster data. Technical Report, downloadable at <http://www.eea.europa.eu/data-and-maps/data/corine-land-cover-2000-raster-1>

### Examples

```
data(turin)
#plot(as.im(turin), main="", col=gray(c(0.8,0)), ribbon=FALSE)

#shannon's entropy
shannon(turin)

#shannon's entropy of Z (urban/non-urban pairs)
shannonZ(turin)

#oneill's entropy
oneill(turin)

#leibovici's entropy only on Collegno's municipality
data(turinTess)
```

```

cell.size=250; ncl=ncol(turin); nrw=nrow(turin)
coords=expand.grid(rev(seq(cell.size/2, (nrw*cell.size-cell.size/2), l=nrw)),
  seq(cell.size/2, (ncl*cell.size-cell.size/2), l=ncl))
data.pp=ppp(x=coords[which(!is.na(c(turin))),2],
  y=coords[which(!is.na(c(turin))),1],
  window=owin(xrange=c(0, ncl*cell.size), yrange=c(0,nrw*cell.size)),
  marks=c(turin)[which(!is.na(c(turin)))]])
data=data.pp[turinTess$tiles[[which(turinTess$names=="Collegno")]]]
#plot(data, pch=16, cex=0.4)
outp=leibovici(data, cell.size=250, ccdist=400, verbose=TRUE)

#altieri's entropy only on Collegno's municipality
outp=altieri(data, cell.size=250, distbreak=c(cell.size, 2*cell.size), verbose=TRUE)

#batty's entropy
#on all points, with a random partition in 10 sub-areas
batty.ent=batty(turin, cell.size=250, partition=10)
#plot with partition
data(turinW)
#plot(as.im(turin, W=turinW), main="", col=gray(c(0.8,0)), ribbon=FALSE)
#plot(batty.ent$areas.tess, add=TRUE, border=2)

#batty's entropy with a partition based on the administrative areas
data(turinTess)
batty.ent=batty(turin, cell.size=250, partition=turinTess)
#plot(as.im(turin, W=turinW), main="", col=gray(c(0.8,0)), ribbon=FALSE)
#for(i in 1:turinTess$n) plot(turinTess$tiles[[i]], add=TRUE, border=2)

#karlstrom and ceccato's entropy
data(turinW)
KC.ent=karlstrom(turin, cell.size=250, partition=15, neigh=3)
#plot with partition
#plot(as.im(turin, W=turinW), main="", col=gray(c(0.8,0)), ribbon=FALSE)
#plot(KC.ent$areas.tess, add=TRUE, border=2)

#karlstrom and ceccato's entropy with a partition based on the administrative areas
data(turinTess)
KC.ent=karlstrom(turin, cell.size=250, partition=turinTess, neigh=5000, method="distance")
#plot(as.im(turin, W=turinW), main="", col=gray(c(0.8,0)), ribbon=FALSE)
#for(i in 1:turinTess$n) plot(turinTess$tiles[[i]], add=TRUE, border=2)

```

---

turinTess

---

*Municipalities' administrative borders for Turin urban data.*


---

## Description

City borders of all municipalities included in the Turin dataset, in the format of polygonal owin objects.

**Usage**

```
turinTess
```

**Format**

A list of three:

- tiles a list of 15, each element is a owin object with the administrative border of one municipality of Turin's dataset
- n the number of municipalities
- names the names of the 15 municipalities, in the same order as the windows

**Details**

This is a list of 15 observation windows created as owin objects based on the coordinates of the border polygons, for each municipality. See ?owin for details. The list also contains the names of the municipalities, in Italian. Examples on the usefulness of the administrative borders can be found at the top [turin](#).

**Source**

EEA (2011). Corine land cover 2000 raster data. Technical Report, downloadable at <http://www.eea.europa.eu/data-and-maps/data/corine-land-cover-2000-raster-1>

**Examples**

```
plot(turinW, col=c("black", "white"), main="")
plot(turinTess$tiles[[1]],border=2, add=TRUE, lwd=2)
for(ll in 2:turinTess$n) plot(turinTess$tiles[[ll]],border=2, add=TRUE, lwd=2)

plot(as.im(turin, W=turinW), main="", col=gray(c(0.8,0)), ribbon=FALSE)
plot(turinTess$tiles[[1]],border=2, add=TRUE, lwd=2)
for(ll in 2:turinTess$n) plot(turinTess$tiles[[ll]],border=2, add=TRUE, lwd=2)

#see examples under the topic "turin"
```

---

```
turinW
```

*Observation window for Turin urban data.*

---

**Description**

An owin object with the city border for the Turin dataset.

**Usage**

```
turinW
```



**Format**

An `owin` object. Units are given in metres; the basic image unit is a 250x250 metres pixels.

**Details**

This observation window is an `owin` object created as a binary mask. See `?owin` for details. Examples on the usefulness of the window can be found at the topic [turin](#).

**Source**

EEA (2011). Corine land cover 2000 raster data. Technical Report, downloadable at <http://www.eea.europa.eu/data-and-maps/data/corine-land-cover-2000-raster-1>

**Examples**

```
plot(turinW, col=c("red", "white"), main="")
plot(as.im(turin, W=turinW), main="", col=gray(c(0.8,0)), ribbon=FALSE, add=TRUE)

#see examples under the topic "turin"
```

---

varshannon

*Estimated variance of Shannon's entropy.*


---

**Description**

This function estimates the variance of Shannon's entropy of a variable  $X$ .

**Usage**

```
varshannon(data)
```

**Arguments**

`data` A data matrix or vector, can be numeric, factor, character, ... Alternatively, a marked ppp object.

**Details**

`varshannon` estimates the variance of the maximum likelihood estimator of Shannon's entropy given by `shannon`. The variance is

$$V(H(X)) = H(X)_2 - H(X)^2$$

, where  $H(X)_2$  is a version of Shannon's entropy (see `shannon`) where the information function  $\log(1/p(x_i))$  is squared:

$$H(X)_2 = \sum p(x_i) \log(1/p(x_i))^2$$

. The function is able to work with lattice data with missing data, as long as they are specified as NAs: missing data are ignored in the computations.

**Value**

the estimated variance of Shannon's entropy.

**Examples**

```
#NON SPATIAL DATA
varshannon(sample(1:5, 50, replace=TRUE))

#POINT DATA
data.pp=runifpoint(100, win=square(10))
marks(data.pp)=sample(c("a","b","c"), 100, replace=TRUE)
varshannon(marks(data.pp))

#LATTICE DATA
data.lat=matrix(sample(c("a","b","c"), 100, replace=TRUE), nrow=10)
varshannon(data.lat)
```

---

varshannonZ

*Estimated variance of Shannon's entropy of Z.*


---

**Description**

This function estimates the variance of Shannon's entropy of  $Z$ , where  $Z$  identifies pairs of categories of the original study variable.

**Usage**

```
varshannonZ(data)
```

**Arguments**

**data** A data matrix or vector, can be numeric, factor, character, ... Alternatively, a marked ppp object.

**Details**

`varshannonZ` estimates the variance of the maximum likelihood estimator of Shannon's entropy of  $Z$  given by `shannonZ`. The variance is

$$V(H(Z)) = H(Z)_2 - H(Z)^2$$

, where

$$H(Z)_2 = \sum p(z_r) \log(1/p(z_r))^2$$

. The function is able to work with lattice data with missing data, as long as they are specified as NAs: missing data are ignored in the computations.

**Value**

the estimated variance of Shannon's entropy of  $Z$ .

**Examples**

```
#NON SPATIAL DATA
data=sample(1:5, 50, replace=TRUE)
varshannonZ(data)

#POINT DATA
data.pp=runifpoint(100, win=square(10))
marks(data.pp)=sample(c("a","b","c"), 100, replace=TRUE)
varshannonZ(marks(data.pp))

#LATTICE DATA
data.lat=matrix(sample(c("a","b","c"), 100, replace=TRUE), nrow=10)
varshannonZ(data.lat)
```

# Index

## \* datasets

raintrees, 15  
raintreesCOV, 17  
turin, 22  
turinTess, 23  
turinW, 24

altieri, 2  
areapart, 4, 6, 10

batty, 6, 10

contagion, 8

karlstrom, 9

leibovici, 11

oneill, 13

parredw, 14

raintrees, 15, 18  
raintreesCOV, 17

shannon, 18, 25  
shannonZ, 3, 19, 26  
shannonZ(), 3  
SpatEntropy, 6, 9, 21

turin, 22, 24, 25  
turinTess, 23  
turinW, 24

varshannon, 25, 25  
varshannonZ, 26, 26