

# Package ‘MRCE’

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**Type** Package

**Title** Multivariate Regression with Covariance Estimation

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**Depends** R (>= 2.10.1), glasso

**Description** Compute and select tuning parameters for the MRCE estimator proposed by Rothman, Levina, and Zhu (2010) <[doi:10.1198/jcgs.2010.09188](https://doi.org/10.1198/jcgs.2010.09188)>. This estimator fits the multiple output linear regression model with a sparse estimator of the error precision matrix and a sparse estimator of the regression coefficient matrix.

**License** GPL-2

**NeedsCompilation** yes

**Repository** CRAN

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MRCE-package	<i>Multivariate regression with covariance estimation</i>
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## Description

Computes the MRCE estimators (Rothman, Levina, and Zhu, 2010) and has the dataset `stock04` used in Rothman, Levina, and Zhu (2010), originally analyzed in Yuan et al. (2007).

## Details

The primary function is [mrce](#). The dataset is [stock04](#).

## Author(s)

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## References

Rothman, A.J., Levina, E., and Zhu, J. (2010). Sparse multivariate regression with covariance estimation. *Journal of Computational and Graphical Statistics* 19:974–962.

Yuan, M., Ekici, A., Lu, Z., and Monteiro, R. (2007). Dimension reduction and coefficient estimation in multivariate linear regression. *Journal of the Royal Statistical Society Series B* 69(3):329–346.

Jerome Friedman, Trevor Hastie, Robert Tibshirani (2008). Sparse inverse covariance estimation with the graphical lasso. *Biostatistics*, 9(3), 432-441.

Jerome Friedman, Trevor Hastie, Robert Tibshirani (2010). Regularization Paths for Generalized Linear Models via Coordinate Descent. *Journal of Statistical Software*, 33(1), 1-22.

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mrce

*Do multivariate regression with covariance estimation (MRCE)*

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## Description

Let  $S_+^q$  be the set of  $q$  by  $q$  symmetric and positive definite matrices and let  $y_i \in R^q$  be the measurements of the  $q$  responses for the  $i$ th subject ( $i = 1, \dots, n$ ). The model assumes that  $y_i$  is a realization of the  $q$ -variate random vector

$$Y_i = \mu + \beta' x_i + \varepsilon_i, \quad i = 1, \dots, n$$

where  $\mu \in R^q$  is an unknown intercept vector;  $\beta \in R^{p \times q}$  is an unknown regression coefficient matrix;  $x_i \in R^p$  is the known vector of values for  $i$ th subjects' predictors, and  $\varepsilon_1, \dots, \varepsilon_n$  are  $n$  independent copies of a  $q$ -variate Normal random vector with mean 0 and unknown inverse covariance matrix  $\Omega \in S_+^q$ .

This function computes penalized likelihood estimates of the unknown parameters  $\mu$ ,  $\beta$ , and  $\Omega$ . Let  $\bar{y} = n^{-1} \sum_{i=1}^n y_i$  and  $\bar{x} = n^{-1} \sum_{i=1}^n x_i$ . These estimates are

$$(\hat{\beta}, \hat{\Omega}) = \arg \min_{(B, Q) \in R^{p \times q} \times S_+^q} \left\{ g(B, Q) + \lambda_1 \left( \sum_{j \neq k} |Q_{jk}| + 1(p \geq n) \sum_{j=1}^q |Q_{jj}| \right) + 2\lambda_2 \sum_{j=1}^p \sum_{k=1}^q |B_{jk}| \right\}$$

and  $\hat{\mu} = \bar{y} - \hat{\beta}' \bar{x}$ , where

$$g(B, Q) = \text{tr}\{n^{-1}(Y - XB)'(Y - XB)Q\} - \log |Q|,$$

$Y \in R^{n \times q}$  has  $i$ th row  $(y_i - \bar{y})'$ , and  $X \in R^{n \times p}$  has  $i$ th row  $(x_i - \bar{x})'$ .

**Usage**

```
mrce(X,Y, lam1=NULL, lam2=NULL, lam1.vec=NULL, lam2.vec=NULL,
     method=c("single", "cv", "fixed.omega"),
     cov.tol=1e-4, cov.maxit=1e3, omega=NULL,
     maxit.out=1e3, maxit.in=1e3, tol.out=1e-8,
     tol.in=1e-8, kfold=5, silent=TRUE, eps=1e-5,
     standardize=FALSE, permute=FALSE)
```

**Arguments**

X	An $n$ by $p$ matrix of the values for the prediction variables. The $i$ th row of $X$ is $x_i$ defined above ( $i = 1, \dots, n$ ). Do not include a column of ones.
Y	An $n$ by $q$ matrix of the observed responses. The $i$ th row of $Y$ is $y_i$ defined above ( $i = 1, \dots, n$ ).
lam1	A single value for $\lambda_1$ defined above. This argument is only used if <code>method="single"</code>
lam2	A single value for $\lambda_2$ defined above (or a $p$ by $q$ matrix with $(j, k)$ th entry $\lambda_{2jk}$ in which case the penalty $2\lambda_2 \sum_{j=1}^p \sum_{k=1}^q  B_{jk} $ becomes $2 \sum_{j=1}^p \sum_{k=1}^q \lambda_{2jk}  B_{jk} $ ). This argument is not used if <code>method="cv"</code> .
lam1.vec	A vector of candidate values for $\lambda_1$ from which the cross validation procedure searches: only used when <code>method="cv"</code> and must be specified by the user when <code>method="cv"</code> . Please arrange in decreasing order.
lam2.vec	A vector of candidate values for $\lambda_2$ from which the cross validation procedure searches: only used when <code>method="cv"</code> and must be specified by the user when <code>method="cv"</code> . Please arrange in decreasing order.
method	There are three options: <ul style="list-style-type: none"> <li>• <code>method="single"</code> computes the MRCE estimate of the regression coefficient matrix with penalty tuning parameters <code>lam1</code> and <code>lam2</code>;</li> <li>• <code>method="cv"</code> performs <code>kfold</code> cross validation using candidate tuning parameters in <code>lam1.vec</code> and <code>lam2.vec</code>;</li> <li>• <code>method="fixed.omega"</code> computes the regression coefficient matrix estimate for which <math>Q</math> (defined above) is fixed at <code>omega</code>.</li> </ul>
cov.tol	Convergence tolerance for the glasso algorithm that minimizes the objective function (defined above) with $B$ fixed.
cov.maxit	The maximum number of iterations allowed for the glasso algorithm that minimizes the objective function (defined above) with $B$ fixed.
omega	A user-supplied fixed value of $Q$ . Only used when <code>method="fixed.omega"</code> in which case the minimizer of the objective function (defined above) with $Q$ fixed at <code>omega</code> is returned.
maxit.out	The maximum number of iterations allowed for the outer loop of the exact MRCE algorithm.
maxit.in	The maximum number of iterations allowed for the algorithm that minimizes the objective function, defined above, with $\Omega$ fixed.
tol.out	Convergence tolerance for outer loop of the exact MRCE algorithm.

tol.in	Convergence tolerance for the algorithm that minimizes the objective function, defined above, with $\Omega$ fixed.
kfold	The number of folds to use when <code>method="cv"</code> .
silent	Logical: when <code>silent=FALSE</code> this function displays progress updates to the screen.
eps	The algorithm will terminate if the minimum diagonal entry of the current iterate's residual sample covariance is less than <code>eps</code> . This may need adjustment depending on the scales of the variables.
standardize	Logical: should the columns of $X$ be standardized so each has unit length and zero average. The parameter estimates are returned on the original unstandardized scale. The default is <code>FALSE</code> .
permute	Logical: when <code>method="cv"</code> , should the subject indices be permuted? The default is <code>FALSE</code> .

### Details

Please see Rothman, Levina, and Zhu (2010) for more information on the algorithm and model. This version of the software uses the `glasso` algorithm (Friedman et al., 2008) through the R package `glasso`. If the algorithm is running slowly, track its progress with `silent=FALSE`. In some cases, choosing `cov.tol=0.1` and `tol.out=1e-10` allows the algorithm to make faster progress. If one uses a matrix for `lam2`, consider setting `tol.in=1e-12`.

When  $p \geq n$ , the diagonal of the optimization variable corresponding to the inverse covariance matrix of the error is penalized. Without diagonal penalization, if there exists a  $\bar{B}$  such that the  $q$ th column of  $Y$  is equal to the  $q$ th column of  $X\bar{B}$ , then a global minimizer of the objective function (defined above) does not exist.

The algorithm that minimizes the objective function, defined above, with  $Q$  fixed uses a similar update strategy and termination criterion to those used by Friedman et al. (2010) in the corresponding R package `glmnet`.

### Value

A list containing

Bhat	This is $\hat{\beta} \in R^{p \times q}$ defined above. If <code>method="cv"</code> , then <code>best.lam1</code> and <code>best.lam2</code> defined below are used for $\lambda_1$ and $\lambda_2$ .
muhat	This is the intercept estimate $\hat{\mu} \in R^q$ defined above. If <code>method="cv"</code> , then <code>best.lam1</code> and <code>best.lam2</code> defined below are used for $\lambda_1$ and $\lambda_2$ .
omega	This is $\hat{\Omega} \in S_+^q$ defined above. If <code>method="cv"</code> , then <code>best.lam1</code> and <code>best.lam2</code> defined below are used for $\lambda_1$ and $\lambda_2$ .
mx	This is $\bar{x} \in R^p$ defined above.
my	This is $\bar{y} \in R^q$ defined above.
best.lam1	The selected value for $\lambda_1$ by cross validation. Will be <code>NULL</code> unless <code>method="cv"</code> .
best.lam2	The selected value for $\lambda_2$ by cross validation. Will be <code>NULL</code> unless <code>method="cv"</code> .
cv.err	Cross validation error matrix with <code>length(lam1.vec)</code> rows and <code>length(lam2.vec)</code> columns. Will be <code>NULL</code> unless <code>method="cv"</code> .

**Note**

The algorithm is fastest when  $\lambda_1$  and  $\lambda_2$  are large. Use `silent=FALSE` to check if the algorithm is converging before the total iterations exceeds `maxit.out`.

**Author(s)**

Adam J. Rothman

**References**

Rothman, A. J., Levina, E., and Zhu, J. (2010) Sparse multivariate regression with covariance estimation. *Journal of Computational and Graphical Statistics*, 19: 947–962.

Jerome Friedman, Trevor Hastie, Robert Tibshirani (2008). Sparse inverse covariance estimation with the graphical lasso. *Biostatistics*, 9(3), 432-441.

Jerome Friedman, Trevor Hastie, Robert Tibshirani (2010). Regularization Paths for Generalized Linear Models via Coordinate Descent. *Journal of Statistical Software*, 33(1), 1-22.

**Examples**

```
set.seed(48105)
n=50
p=10
q=5

Omega.inv=diag(q)
for(i in 1:q) for(j in 1:q)
  Omega.inv[i,j]=0.7^abs(i-j)
out=eigen(Omega.inv, symmetric=TRUE)
Omega.inv.sqrt=tcrossprod(out$vec*rep(out$val^(0.5), each=q),out$vec)
Omega=tcrossprod(out$vec*rep(out$val^(-1), each=q),out$vec)

X=matrix(rnorm(n*p), nrow=n, ncol=p)
E=matrix(rnorm(n*q), nrow=n, ncol=q)%*%Omega.inv.sqrt
Beta=matrix(rbinom(p*q, size=1, prob=0.1)*runif(p*q, min=1, max=2), nrow=p, ncol=q)
mu=1:q

Y=rep(1,n)%*%t(mu) + X*%*Beta + E

lam1.vec=rev(10^seq(from=-2, to=0, by=0.5))
lam2.vec=rev(10^seq(from=-2, to=0, by=0.5))
cvfit=mrce(Y=Y, X=X, lam1.vec=lam1.vec, lam2.vec=lam2.vec, method="cv")
cvfit

fit=mrce(Y=Y, X=X, lam1=10^(-1.5), lam2=10^(-0.5), method="single")
fit

lam2.mat=1000*(fit$Bhat==0)
refit=mrce(Y=Y, X=X, lam2=lam2.mat, method="fixed.omega", omega=fit$omega, tol.in=1e-12)
refit
```

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stock04

*log-returns of 9 stocks from 2004*

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**Description**

Weekly log-returns of 9 stocks from 2004, analyzed in Yuan et al. (2007)

**Usage**

```
data(stock04)
```

**Format**

The format is: num [1:52, 1:9] 0.002275 -0.003795 0.012845 0.017489 -0.000369 ... - attr(\*, "dimnames")=List of 2 ..\$ : NULL ..\$ : chr [1:9] "Walmart" "Exxon" "GM" "Ford" ...

**Source**

Yuan, M., Ekici, A., Lu, Z., and Monteiro, R. (2007). Dimension reduction and coefficient estimation in multivariate linear regression. *Journal of the Royal Statistical Society Series B*, 69(3):329–346.

**References**

Yuan, M., Ekici, A., Lu, Z., and Monteiro, R. (2007). Dimension reduction and coefficient estimation in multivariate linear regression. *Journal of the Royal Statistical Society Series B*, 69(3):329–346.

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